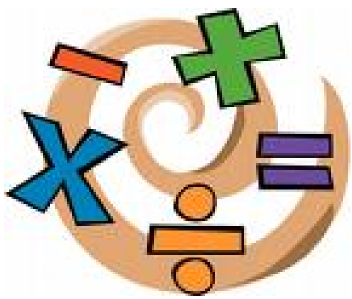
# The Common Core State Standards



## *Considering Student Pathways through the CCSSM*

**David Foster**
*Silicon Valley Mathematics Initiative*
**www.svmimac.org**

# Optimism

"Optimism is an essential ingredient for innovation. How else can the individual welcome change over security, adventure over staying in safe places? A significant innovation has effects that reach much further than can be imagined at the time, and creates its own uses. It will not be held back by those who lack the imagination to exploit its use, but will be swept along by the creative members of our society for the good of all. Innovation cannot be mandated any more than a baseball coach can demand that the next batter hit a home run. He can, however, assemble a good team, encourage his players, and play the odds."

*Robert N. Noyce*

# Silicon Valley Mathematics Initiative

## *Approximately 100 Members - School Districts, Charter School Networks, and Schools*

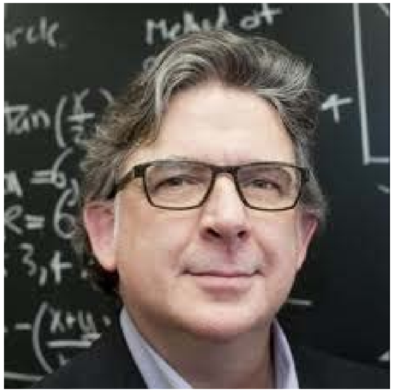
- Albany USD
- Alvord SD (Riverside County)
- Antioch Unified SD
- Aspire Charter School Network
- Assumption School
- Atlanta PS
- Bayshore SD
- Belmont-Redwood Shores SD
- Berryessa SD
- Bolinas – Lagunitas SD
- Brisbane SD
- Buckeye SD
- Cambrian SD
- Campbell Union HSD
- Campbell Union ESD
- Castro Valley USD
- Carmel HS
- Charter Oak USD
- Charter School of Morgan Hill
- Chicago Public School
- Creative Arts Charter (SF)
- Cristo Rey Network
- CSU San Bernardino
- Cotati – Rohnert Park
- Cupertino SD
- Dade County Schools (GA)
- Del Mar USD (San Diego Co)
- Discovery Charter School
- Dioceses of Santa Clara
- Dublin USD
- East Side UHSD
- Edmonds Community College
- Emery SD
- Etiwanda SD
- Evergreen SD
- Gilroy USD
- Fairfield-Suisun USD
- Franklin-McKinley SD
- Fremont Union HSD
- Fremont USD
- Forsyth County School (GA)
- Half Moon Bay
- Hamilton County (Tn)
- Hayward USD
- Jefferson ESD
- Jefferson HSD
- Las Lomitas SD
- La Honda-Pescadero Sd
- Live Oak School Districts
- Livermore USD
- Los Altos SD
- Los Gatos SD
- Menlo Park SD
- Monterey Peninsula USD
- Moreland SD
- Moraga SD
- Morgan Hill USD
- Mountain SD
- Mountain View SD
- National Council of La Raza
- New York City PS
- New Visions for Public Schools
- Oakland Unified SD
- Pacific Grove HS
- Oak Grove SD
- Pacifica SD
- Pajaro Valley USD
- Palo Alto USD
- Pittsburgh USD
- Portola Valley SD
- Ravenswood City SD
- Riverside COE
- Redwood City Schools
- Sacramento City USD
- Salinas City Schools
- San Bruno Park
- San Carlos CLC
- San Diego COE
- San Francisco USD
- SMFC (Park School)
- San Jose Unified SD
- San Mateo UHSD
- San Ramon Valley USD
- Santa Ana USD
- Santa Clara USD
- Santa Cruz CSD
- Saint Michael's School (Poway)
- Saint Patrick's School (San Jose)
- Saratoga
- Scotts Valley USD
- Santa Clara COE
- Sequoia HSD
- SMCOE County Court Schools
- South Cook Service District
- South San Francisco USD
- Sumter County (GA)
- Tehama COE
- The Nueva School
- Union SD
- University of Illinois, Chicago
- Valley Christen (Dublin)
- Valdosta City (GA)
- Walnut Creek SD
- Woodside SD

# Common Core Standards:

## *A New Direction linking Instruction and Assessment*

# Three Central Authors

## *Common Core State Standards in Mathematics*



Bill McCallum

Phil Daro

Jason Zimba

Charges given to the authors:

- All students College and Career Ready by 11th grade
- Internationally Benchmarked
- Make the standards “Fewer, Clear and Higher”

# CCSS Mathematical Practices

**OVERARCHING HABITS OF MIND**

1. Make sense of problems and persevere in solving them
6. Attend to precision

**REASONING AND EXPLAINING**

2. Reason abstractly and quantitatively
3. Construct viable arguments and critique the reasoning of others

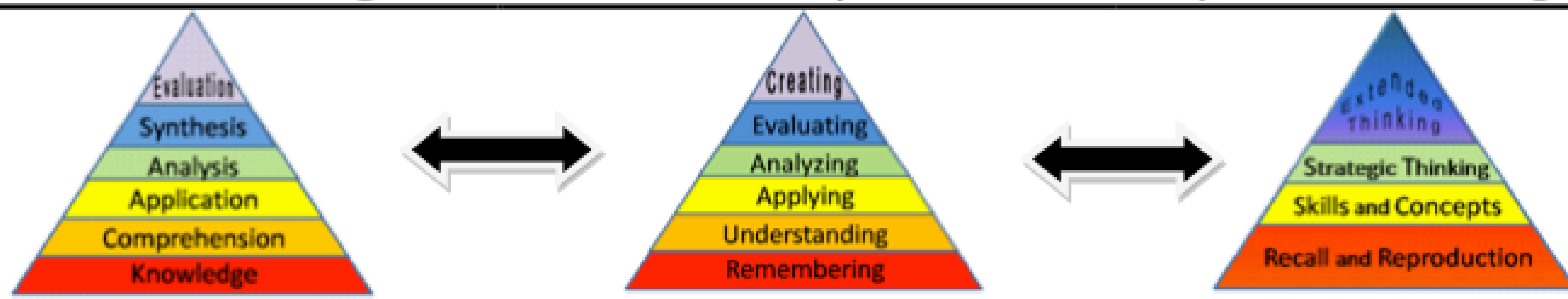
**MODELING AND USING TOOLS**

4. Model with mathematics
5. Use appropriate tools strategically

**SEEING STRUCTURE AND GENERALIZING**

7. Look for and make use of structure
8. Look for and express regularity in repeated reasoning

# Levels of Thinking in Bloom's Taxonomy and Webb's Depth of Knowledge




Bloom's – Old Version (1956) | Bloom's - New Version (1990's) | Webb's DOK (2002)

*Bloom's six major categories were changed from noun to verb forms in the new version which was developed in the 1990's and released in 2001. The knowledge level was renamed as remembering. Comprehension was retitled understanding, and synthesis was renamed as creating. In addition, the top two levels of Bloom's changed position in the revised version.*

| Bloom's Taxonomy | Revised Bloom's Taxonomy |
|---|---|
| **Knowledge** | **Remembering** |
| *Recall appropriate information.* | |
| **Comprehension** | **Understanding** |
| *Grasp the meaning of material.* | |
| **Application** | **Applying** |
| *Use learned material in new and concrete situations.* | |
| **Analysis** | **Analyzing** |
| *Break down material into component parts so that its organizational structure may be understood.* | |
| **Synthesis** | **Evaluating** |
| *Put parts together to form a new whole.* | *Make judgments based on criteria and standards.* |
| **Evaluation** | **Creating** *(Previously Synthesis)* |
| *Judge value of material for a given purpose.* | *Put elements together to form a coherent or functional whole; reorganizing elements into a new pattern or structure through generating, planning, or producing.* |

Norman L. Webb of Wisconsin Center for Educational Research generated DOK levels to aid in alignment analysis of curriculum, objectives, standards, and assessments.

## Webb's Depth of Knowledge & Corresponding Verbs

**Some verbs could be classified at different levels depending on application.*

### Recall and Reproduction *Correlates to Bloom's 2 Lowest Levels*

*Recall a fact, information, or procedure.*

arrange, calculate, define, draw, identify, list, label, illustrate, match, measure, memorize, quote, recognize, repeat, recall, recite, state, tabulate, use, tell who- what- when- where- why

### Skill/Concept

*Engages mental process beyond habitual response using information or conceptual knowledge. Requires two or more steps.*

apply, categorize, determine cause and effect, classify, collect and display, compare, distinguish, estimate, graph, identify patterns, infer, interpret, make observations, modify organize, predict, relate, sketch, show, solve, summarize, use context clues

### Strategic Thinking

*Requires reasoning, developing plan or a sequence of steps, some complexity, more than one possible answer, higher level of thinking than previous 2 levels.*

apprise, assess, cite evidence, critique, develop a logical argument, differentiate, draw conclusions, explain phenomena in terms of concepts, formulate, hypothesize, investigate revise, use concepts to solve non-routine problems

### Extended Thinking *Correlates to Bloom's 2 Highest Levels*

*Requires investigation, complex reasoning, planning, developing, and thinking-probably over an extended period of time. *Longer time period is not an applicable factor if work is simply repetitive and/or does not require higher-order thinking.*

analyze, apply concepts, compose, connect, create, critique, defend, design, evaluate, judge, propose, prove, support, synthesize

# Depth of Knowledge (DOK)

*Low-Cognitive Demand*

## Level 1: Recalling and Recognizing

Student is able to recall routine facts of knowledge and can recognize shape, symbols, attributes and other qualities.

## Level 2: Using Procedures

Student uses or applies procedures and techniques to arrive at solutions or answers.

# Depth of Knowledge (DOK)

## *High-Cognitive Demand*

## Level 3: Explaining and Concluding

Student reasons and derives conclusions. Student explains reasoning and processes. Student communicates procedures and findings.

## Level 4: Making Connections, Extending and Justifying

Student makes connections between different concepts and strands of mathematics. Extends and builds on knowledge to a situation to arrive at a conclusion. Students use reason and logic to prove and justify conclusions.

# Common Core Big Ideas
# Depth of Knowledge (DOKs)

| | Mathematics | | ELA/Literacy | |
|---|---|---|---|---|
| | DOK3 | DOK4 | DOK3 | DOK4 |
| Current Assessments | <2% | 0% | 20% | 2% |
| New SBAC Assessments | 49% | 21% | 43% | 25% |

Yuan & Le (2012); Herman & Linn (2013) from Linda Darling-Hammond, Assembly Hearing, 3.6.13

# Goals of Assessment

"We must ensure that tests measure what is of value, not just what is easy to test. If we want students to investigate, explore, and discover, assessment must not measure just mimicry mathematics."

Everybody Counts

# CST – Released Items Algebra 1

**The total cost ($c$) in dollars of renting a sailboat for $n$ days is given by the equation**

$$c = 120 + 60n.$$

**If the total cost was $360, for how many days was the sailboat rented?**

**A** 2

**B** 4

**C** 6

**D** 8

# SMARTER BALANCE Assessment Consortia



MARS

MARS Team

Mathematics Assessment Resource Service

Stanford University

School of Education

**Developed Content Specifications for SBAC**

# Content Specifications
# for the Summative assessment of the
# *Common Core State Standards for Mathematics*

## DRAFT TO ACCOMPANY GOVERNING STATE VOTE ON ASSESSMENT CLAIMS

**March 20, 2012**

**Developed with input from content experts and Smarter Balanced Assessment Consortium Staff, Work Group Members, and Technical Advisory Committee**

# Acknowledgements

**Alan Schoenfeld**, University of California at Berkeley and **Hugh Burkhardt**, Shell Centre, University of Nottingham served as principal authors of this paper. Sections of the document were also authored by **Jamal Abedi**, University of California at Davis; **Karin Hess**, National Center for the Improvement of Educational Assessment; **Martha Thurlow**, National Center on Educational Outcomes, University of Minnesota

Significant contributions and organization of this second draft were provided by **Shelbi Cole**, Connecticut State Department of Education, and **Jason Zimba**, Student Achievement Partners. The project was facilitated by **Linda Darling-Hammond** at Stanford University.

Others who offered advice and feedback on the document include:

**Rita Crust,** Lead Designer, Mathematics Assessment Resource Service
Past President, Association of State Supervisors of Mathematics

**Brad Findell**, Former Mathematics Initiatives Administrator, Ohio Department of Education

**David Foster,** Director, Silicon Valley Mathematics Initiative

**Henry Pollak,** Adjunct Professor, Columbia University, Teachers College,
Former Head of Mathematics and Statistics, Bell Laboratories

**W. James Popham,** Emeritus Professor, University of California, Los Angeles

**Cathy Seeley,** Senior Fellow, Charles A. Dana Center, The University of Texas at Austin

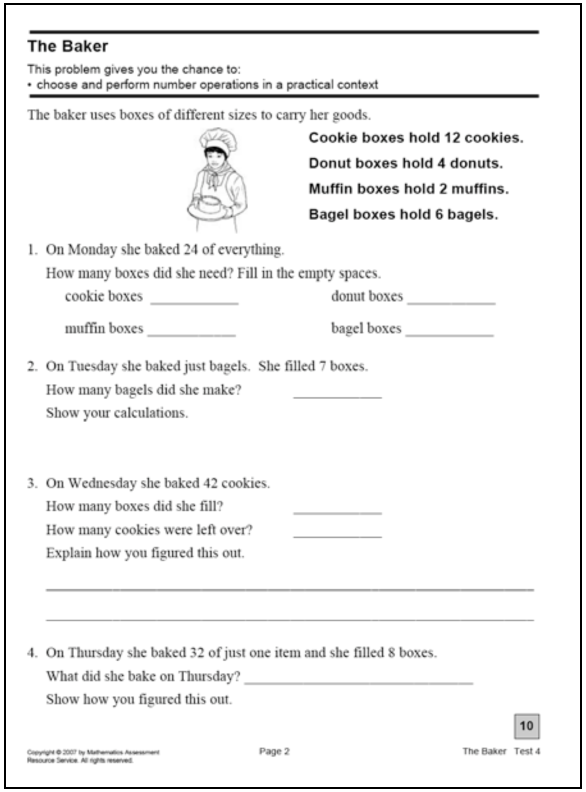
**Malcolm Swan,** Professor of Mathematics Education, Centre for Research in Mathematic Education,
University of Nottingham

# Claims

| Smarter Balanced |
|---|
| 1. Concepts and Procedures: Students can explain and apply mathematical concepts and interpret and carry out mathematical procedures with precision and fluency. |
| 2. Problem Solving: Students can solve a range of complex well-posed problems in pure and applied mathematics, making productive use of knowledge and problem solving strategies. |
| 3. Communicating Reasoning: Students can clearly and precisely construct viable arguments to support their own reasoning and to critique the reasoning of others. |
| 4. Modeling and Data Analysis: Students can analyze complex, real-world scenarios and can construct and use mathematical models to interpret and solve problems. |

# Performance Assessments

*To Inform Instruction And Measure Higher Level Thinking*


**The Baker**

This problem gives you the chance to:
• choose and perform number operations in a practical context

The baker uses boxes of different sizes to carry her goods.

**Cookie boxes hold 12 cookies.**
**Donut boxes hold 4 donuts.**
**Muffin boxes hold 2 muffins.**
**Bagel boxes hold 6 bagels.**

1. On Monday she baked 24 of everything.
   How many boxes did she need? Fill in the empty spaces.
   cookie boxes __________ donut boxes __________
   muffin boxes __________ bagel boxes __________

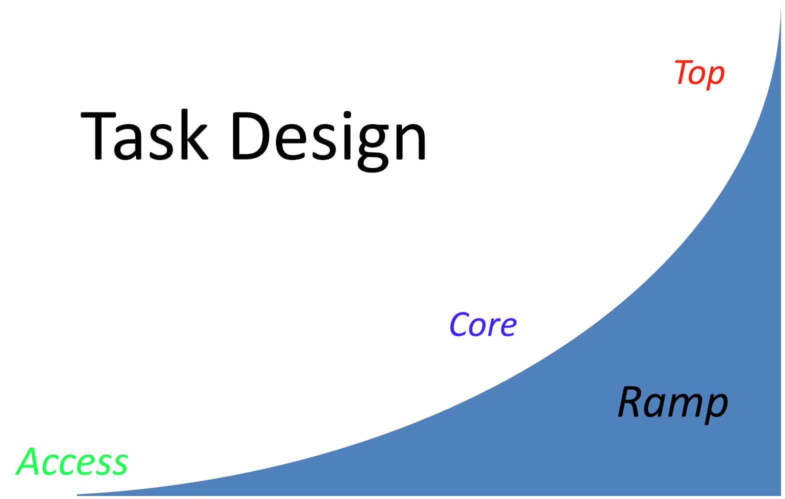
2. On Tuesday she baked just bagels. She filled 7 boxes.
   How many bagels did she make? __________
   Show your calculations.

3. On Wednesday she baked 42 cookies.
   How many boxes did she fill? __________
   How many cookies were left over? __________
   Explain how you figured this out.

4. On Thursday she baked 32 of just one item and she filled 8 boxes.
   What did she bake on Thursday? ____________________
   Show how you figured this out.

10




**Entry level (access into task)**

**Core Mathematics - (meeting standards)**

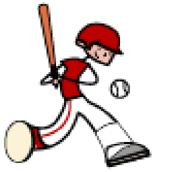
**Top of Ramp (conceptually deeper, beyond)**

- The Mathematics Assessment Resource Service (MARS) is an NSF funded collaboration between U.C. Berkeley and the Shell Centre in Nottingham England.
- The Assessments target grades 2- Geometry and are aligned with the State and NCTM National Math Standards.

SHELL CENTRE BALANCED ASSESSMENT MARS

# Apprentice Task

## CR 4: Baseball Jerseys



Bill is going to order new jerseys for his baseball team.

The jerseys will have the team logo printed on the front.

Bill asks 2 local companies to give him a price.

1. 'Print It' will charge $21.50 each for the jerseys.

   Using *n* for the number of jerseys ordered and *c* for the total cost in dollars, write an equation to show the total cost of jerseys from 'Print It'.

   ______________

2. 'Top Print' has a Set-Up cost of $70 and then charges $18 for each jersey.

   Using *n* to stand for the number of jerseys ordered and *c* for the total cost in dollars, write an equation to show the total cost of jerseys from 'Top Print'.

   ______________

3. Use the two equations from questions 1 and 2 to figure out how many jerseys Bill would need to order for the price from 'Top Print' to be less than from 'Print It'.
   Explain how you figured it out.

   ______________

   ________________________________________________

   ________________________________________________

   ________________________________________________

   ________________________________________________

4. Bill decides to order 30 jerseys from 'Top Print'.
   How much more would the jerseys have cost if he had bought them from 'Print It'?
   Show all your calculations.

   ______________

# Baseball Jerseys

This problem gives you the chance to:
- work with equations that represent real life situations

Bill is going to order new jerseys for his baseball team.

The jerseys will have the team logo printed on the front.

Bill asks two local companies to give him a price.

1. 'Print It' will charge $21.50 each for the jerseys.

   Using *n* for the number of jerseys ordered, and *c* for the total cost in dollars, write an equation to show the total cost of jerseys from 'Print It'.

   ______________________

2. 'Top Print' has a one-time setting up cost of $70 and then charges $18 for each jersey.

   Using *n* to stand for the number of jerseys ordered, and *c* for the total cost in dollars, write an equation to show the total cost of jerseys from 'Top Print'.

   ______________________

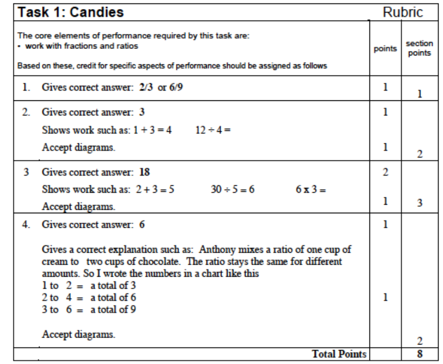
3. Bill decides to order 30 jerseys from 'Top Print'.
How much more would the jerseys cost if he buys them from 'Print It'?
Show all your calculations. ____________________

4. Use the two equations from questions 1 and 2 to figure out how many jerseys Bill would need to buy for the price from 'Top Print' to be less than from 'Print It'.
Explain how you figured it out. ____________________

# Performance Exams

40,000 – 70,000 students per year since 1999

Students in grades 2 through $10^{th}/11^{th}$ grade are administered performance exams (5 apprentice tasks per exam).



| Task 1: Candies | Rubric | |
|---|---|---|
| The core elements of performance required by this task are:<br>• work with fractions and ratios<br>Based on these, credit for specific aspects of performance should be assigned as follows | points | section points |
| 1. Gives correct answer: **2/3** or **6/9** | 1 | 1 |
| 2. Gives correct answer: **3**<br>Shows work such as: 1 + 3 = 4 12 ÷ 4 =<br>Accept diagrams. | 1<br><br>1 | 2 |
| 3 Gives correct answer: **18**<br>Shows work such as: 2 + 3 = 5 30 ÷ 5 = 6 6 x 3 =<br>Accept diagrams. | 2<br><br>1 | 3 |
| 4. Gives correct answer: **6**<br>Gives a correct explanation such as: Anthony mixes a ratio of one cup of cream to two cups of chocolate. The ratio stays the same for different amounts. So I wrote the numbers in a chart like this<br>1 to 2 = a total of 3<br>2 to 4 = a total of 6<br>3 to 6 = a total of 9<br>Accept diagrams. | 1<br><br>1 | 2 |
| **Total Points** | | **8** |

District scoring leaders are trained in using task specific rubrics

Student tests are hand scored by classroom teachers trained and calibrated using standard protocols.

Random sample of student papers are audited and rescored by SJSU math & CS students. (Two reader correlation >0.95)

Educational Data Systems

Student results are collected, analyzed, and reported by an independent data contractor.

# MAC vs. CST 2012

Silicon Valley Mathematics Initiative

*Mathematics Assessment Collaborative*

*Performance Assessment Exam 2012*

# What can MARS tests tell us?

| | Below standards on MARS test | Meeting/exceeding on MARS test |
|---|---|---|
| Below standards on NCLB test | Accurately identified as struggling | |
| Meeting/exceeding on NCLB test | | Accurately identified as understanding |

# What can MARS tests tell us?

| | Below standards on MARS test | Meeting/exceeding on MARS test |
|---|---|---|
| Below standards on NCLB test | Accurately identified as struggling | Misidentified as struggling (“hidden gems”) |
| Meeting/exceeding on NCLB test | Misidentified as understanding (“false positives”) | Accurately identified as understanding |

# MAC vs. CST 2012

Silicon Valley Mathematics Initiative

*Mathematics Assessment Collaborative*

*Performance Assessment Exam 2012*

# MAC vs CST 2012

| 2nd Grade | MAC Level 1 | MAC Level 2 | MAC Level 3 | MAC Level 4 |
|---|---|---|---|---|
| Far Below Basic | 1.0% | 0.3% | 0.1% | 0.0% |
| Below Basic | 1.9% | 2.4% | 1.2% | 0.0% |
| Basic | 1.3% | 4.8% | 5.5% | 0.3% |
| Proficient | 0.4% | 3.5% | 17.7% | 3.4% |
| Advanced | 0.3% | 0.9% | 23.4% | 31.4% |

| 2nd Grade | MAC Below | MAC At / Above | Total |
|---|---|---|---|
| CST Below | 11.7% | 7.1% | 18.8% |
| CST At / Above | 5.1% | 75.9% | 81.0% |
| Total | 16.8% | 83.0% | 100% |

# Elementary Grades

| 3rd Grade | MAC Below | MAC At / Above | Total |
|---|---|---|---|
| CST Below | 15.9% | 5.2% | 21.1% |
| CST At / Above | 13.7% | 65.4% | 79.1% |
| Total | 29.6% | 70.6% | 100% |

| 4th Grade | MAC Below | MAC At / Above | Total |
|---|---|---|---|
| CST Below | 16.9% | 2.8% | 19.7% |
| CST At / Above | 20.3% | 60.0% | 80.3% |
| Total | 37.2% | 62.8% | 100% |

| 5th Grade | MAC Below | MAC At / Above | Total |
|---|---|---|---|
| CST Below | 20.6% | 3.8% | 24.4% |
| CST At / Above | 18.7% | 56.9% | 75.6% |
| Total | 39.3% | 60.7% | 100% |

# Middle School

| 6th Grade | MAC Below | MAC At / Above | Total |
|---|---|---|---|
| CST Below | 37.2% | 1.4% | 38.6% |
| CST At / Above | 25.1% | 36.5% | 61.6% |
| Total | 62.3% | 37.9% | 100% |

| 7th Grade | MAC Below | MAC At / Above | Total |
|---|---|---|---|
| CST Below | 33.3% | 2.1% | 35.4% |
| CST At / Above | 27.4% | 37.1% | 64.5% |
| Total | 60.7% | 39.2% | 100% |

| Course 1 | MAC Below | MAC At / Above | Total |
|---|---|---|---|
| CST Below | 34.5% | 3.6% | 38.1% |
| CST At / Above | 30.3% | 31.5% | 61.8% |
| Total | 64.8% | 35.1% | 100% |

# 8th Graders Taking HS Geometry

| Course 2 | MAC Below | MAC At/Above | Total |
|---|---|---|---|
| CST Below | 3.1% | 0.8% | 3.9% |
| CST At/Above | 51.3% | 44.8% | 96.1% |
| Total | 54.4% | 45.6% | 100% |

# Domains K–8

| K | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Counting & Cardinality | | | | | | Ratios & Proportional Relationships | | |
| Operations and Algebraic Thinking | | | | | | The Number System | | |
| Number and Operations in Base Ten | | | | | | Expressions and Equations | | |
| | | | Fractions | | | | | Functions |
| Measurement and Data | | | | | | | | |
| Geometry | | | | | | Geometry | | |
| | | | | | | Statistics and Probability | | |

# Mathematics Standards for High School

Arranged by conceptual cluster (NOT by course):

- Number and Quantity
- Algebra
- Functions
- Modeling
- Geometry
- Statistics & Probability

# Two Mathematics Pathways

## Two Regular Sequences:

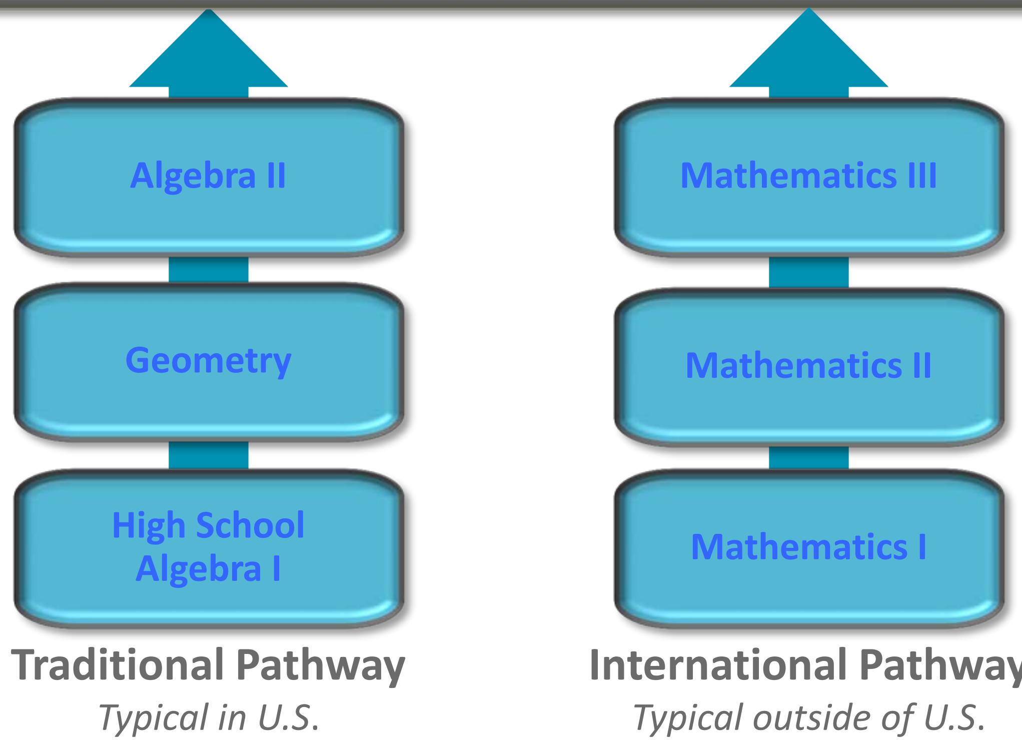
### *Traditional Pathway*

- 2 Algebra courses,1 Geometry course, with Probability and Statistics interwoven

### *International Pathway*

- 3 courses that attend to Algebra, Geometry, and Probability and Statistics each year

Courses in higher level mathematics: Precalculus, Calculus*, Advanced Statistics, Discrete Mathematics, Advanced Quantitative Reasoning, or courses designed for career technical programs of study.



# Credentialing

- Multiple Subject Credential with a Supplementary Authorization
  - Can only teach mathematics to students in grades 9 and below
  - Can teach any mathematics content

- Single Subject Teaching Credential with a Math Supplementary
  - Can teach mathematics to students in grades K-12
  - Mathematics content is from grade 9 or below courses

- Subject Matter Authorization
  - Can teach mathematics to students in grades K-12
  - Mathematics content is from grade 9 or below courses

# Credentialing (continued)

- Single Subject Teaching Credential-Foundational Level Mathematics
  - Can teach
    - General mathematics
    - All levels of Geometry
    - Probability and Statistics
    - Consumer Mathematics
  - Cannot teach
    - Trigonometry (unless it's being introduced in one of the above listed courses)
    - Calculus
    - Math Analysis
  - Can be taught to students in grades K-12

# Credentialing (continued)

- Single Subject Teaching- Mathematics
  - Can teach mathematics to students in grades K-12
  - Can teach the following mathematics courses:
    - General mathematics
    - All levels of Geometry
    - Probability and Statistics
    - Consumer Mathematics
    - Trigonometry
    - Pre-Calculus
    - Math Analysis
    - Calculus

# A-G Requirements

**Board of Admissions and Relations with Schools (BOARS)**
**Statement on High School Mathematics Curriculum Development under the Common Core State Standards**

April 2013

Consistent with past policy and practice for course approval, BOARS reiterates its full support for either the integrated pathways or the traditional pathways, as stated in the A-G Guide's section on Mathematics ("c"). It is BOARS' expectation that courses developed in accordance with either sequence will receive subject area "c" approval provided that they satisfy the course requirements for area "c" presented in the A-G Guide and that they support students in achieving the Standards of Mathematical Practice given in the CCSSM:

http://senate.universityofcalifornia.edu/committees/boars/BOARSonCCSSMathCourseDevelopment.pdf

# Algebra Forever vs CCSSM

Arnold Schwarzenegger
July 8, 2008

"We have made significant gains in enrolling students in Algebra I in eighth grade in recent years, surpassing other state in the U.S. But we must set our goal higher."

We have also made more significant gains in FAILING students in Algebra I in eighth grade in recent years, surpassing other state in the U.S.

3 out of 4 failed in 2008

California Adopted the CCSSM on August 2, 2010 with an addition 15% of a traditional Algebra 1 course and other added standards. We selected PARCC as the assessment to complete the Race to the Top application that we never won.

CALIFORNIA DEPARTMENT OF EDUCATION

# NEWS RELEASE

TOM TORLAKSON
State Superintendent
of Public Instruction

## California Adopts Modified Math Standards to Restore Local Decision Making

*Required by Legislation, Move Allows Progress Toward Common Core*

The move rescinds action by the prior Board in 2010, which adopted standards that contained a unique Grade 8 Algebra I course inconsistent with the published *Common Core State Standards for Mathematics*.

Torlakson recommended the unique Grade 8 Algebra I course be replaced with Algebra I and Mathematics I courses based upon the *Common Core State Standards for Mathematics*.

**Date:** Wed, 16 Jan 2013

# *The California Algebra Experiment*

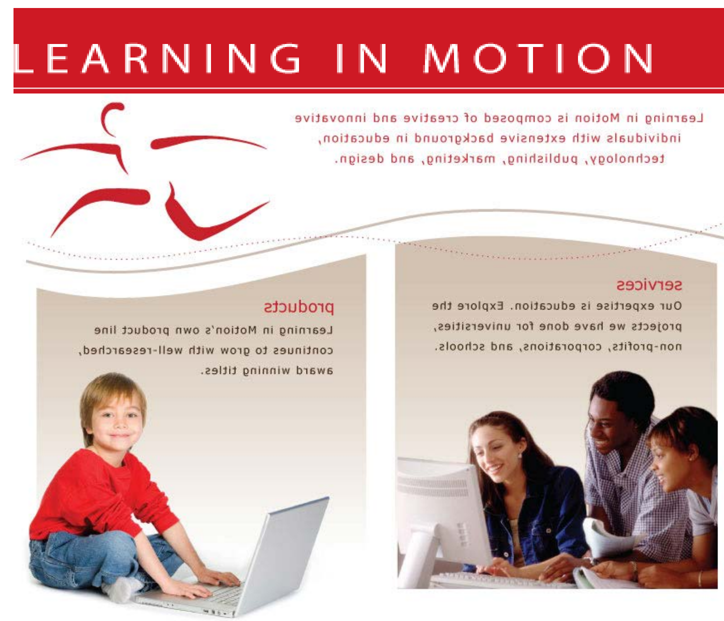
- In 2012, 59% of all eighth grade students took the CST Algebra 1 exam and more than half were not successful. Even more will repeat the class again in high school.
- In 9th grade, 49% of the students took CST Algebra 1 exam and 75% of those students did not pass.
- Research studies indicate nearly 65% of the students who were placed in Algebra in eighth grade are placed in the same level of Algebra in ninth grade.
- About 46% of the students who were successful in Algebra in the eighth grade (B- grade and Proficient) and who were placed again in Algebra in ninth grade were less successful in their second experience.

**It is not Algebra for All, it is Algebra Forever.**

# New K-12 Math Curriculum Inspired by The Common Core State Standards

BILL & MELINDA GATES foundation

PEARSON FOUNDATION

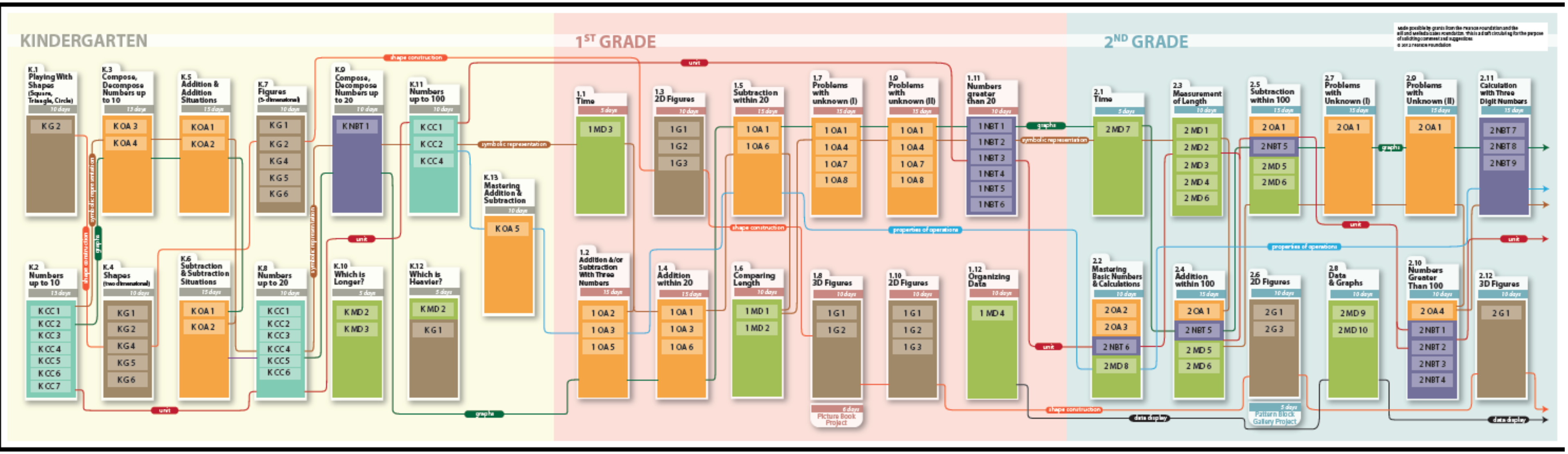



The Gates Foundation and the Pearson Foundation are funding a large scale project to create a system of courses to support the ELA and Mathematics CCSS. These will be a modular, electronic curriculum spanning all grade levels. A Santa Cruz based company, Learning In Motion, is working to write the lessons.

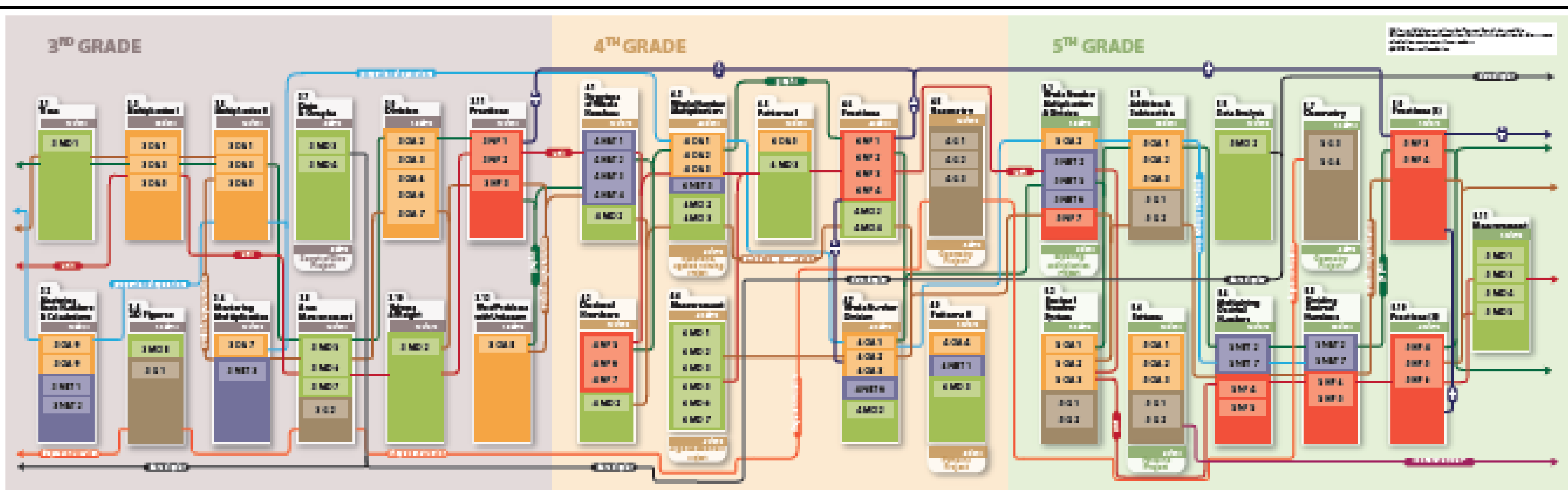
# Think in Terms of Units

Phil Daro has suggested that it is not the lesson or activity, but rather the **unit** that is the "optimal grain-size for the learning of mathematics". Hence that was the starting point for our Scope and Sequence.

Developers of High School: Patrick Callahan, Dick Stanley, David Foster, Brad Findell, Phil Daro, and Marge Cappo

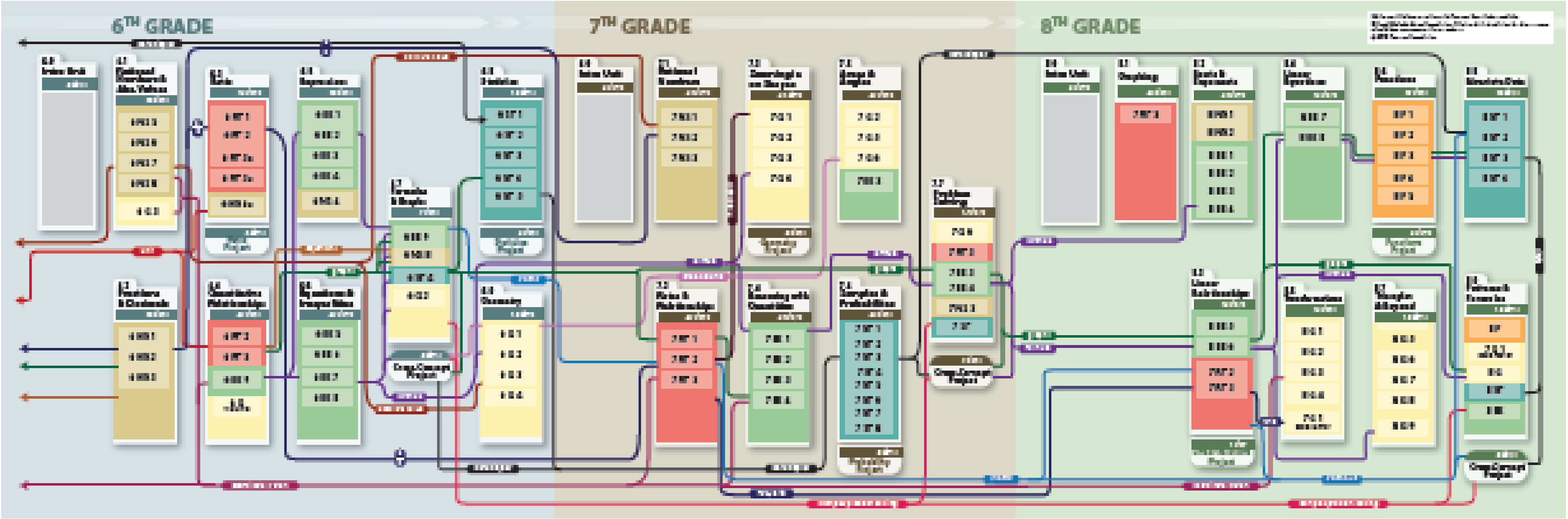


KINDERGARTEN
1ST GRADE
2ND GRADE
K.1 Playing With Shapes (Square, Triangle, Circle)
10 days
KG2
K.3 Compose, Decompose Numbers up to 10
15 days
KOA3
KOA4
K.5 Addition & Addition Situations
15 days
KOA1
KOA2
K.7 Figures (3-dimensional)
10 days
KG1
KG2
KG4
KG5
KG6
K.9 Compose, Decompose Numbers up to 20
10 days
KNBT1
K.11 Numbers up to 100
10 days
KCC1
KCC2
KCC4
K.13 Mastering Addition & Subtraction
10 days
KOA5
K.2 Numbers up to 10
15 days
KCC1
KCC2
KCC3
KCC4
KCC5
KCC6
KCC7
K.4 Shapes (two dimensional)
10 days
KG1
KG2
KG4
KG5
KG6
K.6 Subtraction & Subtraction Situations
15 days
KOA1
KOA2
K.8 Numbers up to 20
10 days
KCC1
KCC2
KCC3
KCC4
KCC5
KCC6
K.10 Which is Longer?
5 days
KMD2
KMD3
K.12 Which is Heavier?
5 days
KMD2
KG1
1.1 Time
5 days
1MD3
1.3 2D Figures
10 days
1G1
1G2
1G3
1.5 Subtraction within 20
15 days
1OA1
1OA6
1.7 Problems with unknown (I)
15 days
1OA1
1OA4
1OA7
1OA8
1.9 Problems with unknown (II)
15 days
1OA1
1OA4
1OA7
1OA8
1.11 Numbers greater than 20
10 days
1NBT1
1NBT2
1NBT3
1NBT4
1NBT5
1NBT6
1.2 Addition &/or Subtraction With Three Numbers
15 days
1OA2
1OA3
1OA5
1.4 Addition within 20
15 days
1OA1
1OA3
1OA6
1.6 Comparing Length
10 days
1MD1
1MD2
1.8 3D Figures
10 days
1G1
1G2
6 days
Picture Book Project
1.10 2D Figures
10 days
1G1
1G2
1G3
1.12 Organizing Data
10 days
1MD4
2.1 Time
5 days
2MD7
2.3 Measurement of Length
10 days
2MD1
2MD2
2MD3
2MD4
2MD6
2.5 Subtraction within 100
15 days
2OA1
2NBT5
2MD5
2MD6
2.7 Problems with Unknown (I)
15 days
2OA1
2.9 Problems with Unknown (II)
15 days
2OA1
2.11 Calculation with Three Digit Numbers
15 days
2NBT7
2NBT8
2NBT9
2.2 Mastering Basic Numbers & Calculations
10 days
2OA2
2OA3
2NBT6
2MD8
2.4 Addition within 100
15 days
2OA1
2NBT5
2MD5
2MD6
2.6 2D Figures
10 days
2G1
2G3
5 days
Pattern Block Gallery Project
2.8 Data & Graphs
10 days
2MD9
2MD10
2.10 Numbers Greater Than 100
10 days
2OA4
2NBT1
2NBT2
2NBT3
2NBT4
2.12 3D Figures
10 days
2G1
shape construction
unit
symbolic representation
graphs
properties of operations
data display

3RD GRADE
4TH GRADE
5TH GRADE

# Middle School Curriculum



# CCSS High School Units

High School Algebra Units:
A0 Introductory Unit
A1 Modeling with Functions
A2 Linear Functions
A3 Linear Equations and Ineq in One Var
A4 Linear Equations and Ineq in Two Var
A5 Quadratic Functions
A6 Quadratic Equations
A7 Exponential Functions
A8 Trigonometric Functions
A9 Functions
A10 Rational and Polynomial Expressions

High School Geometry Units:
G0 Introduction and Construction
G1 Basic Definitions and Rigid Motions
G2 Geometric Relationships and Properties
G3 Similarity
G4 Coordinate Geometry
G5 Circle and Conics
G6 Trigonometric Ratios
G7 Geometric Measurement and Dimension
M4 Capstone Geometric Modeling Project

High School Prob & Stat Units:
P1 Probability
S1 Statistics
S2 Statistics (Random Process)

TRADITIONAL US

## Grade 9: Algebra One

TRADITIONAL

- **A0 Intro Unit** — 5 days
- **A1 Modeling with Functions** — 15 days: N-Q 1, N-Q 2, N-Q 3, F-IF 1, F-IF 2, F-IF 3, F-IF 4*, F-IF 5*, F-IF 9*, F-LE 1, F-LE 1a,1b, F-LE 3, F-LE 5
- **A2 Linear Functions** — 15 days: F-IF 6, F-IF 7a, F-IF 9, F-BF 1, F-BF 1a, F-BF 2, F-BF 4a, F-LE 1, F-LE 1a,1b
- **A3 Linear Equations & Inequalities in One Variable** — 15 days: A-REI 1, A-REI 3, A-REI 11*, A-CED 1, A-CED 3, A-CED 4
- **Modeling Unit** — 4 days
- **A4 Linear Equations & Inequalities in Two Variables** — 15 days: A-CED 2, A-CED 3, A-CED 4, A-REI 5, A-REI 6, A-REI 10, A-REI 12
- **P1 PROJECT** — 5 days
- **A5 Quadratic Functions** — 20 days: F-IF 4*, F-IF 5*, F-IF 6, F-IF 7, F-IF 7a*, F-IF 8, F-IF 8a, F-IF 9, F-BF 1, F-BF 1a, b, F-BF 3
- **A6 Quadratic Equations** — 26 days: N-CN 7, A-SSE 3a, A-SSE 3b, A-REI 4, A-REI 4a,b, A-REI 7
- **Modeling Unit** — 4 days
- **S1 Statistics** — 30 days: S-ID 1, S-ID 2, S-ID 3, S-ID 5, S-ID 6, S-ID 6a, b, c, S-ID 7, S-ID 8, S-ID 9
- **P2 PROJECT** — 5 days

## Grade 10: Geometry

TRADITIONAL

- **G0 Intro and Construction** — 12 days: G-CO 12, G-CO 13
- **G1 Basic Definitions & Rigid Motions** — 20 days: G-CO 1, G-CO 3, G-CO 2, G-CO 4, G-CO 5, G-CO 6, G-CO 7, G-CO 8
- **G2 Geometric Relationships & Properties** — 15 days: G-CO 9, G-CO 10, G-CO 11, G-C 3
- **G3 Similarity** — 20 days: G-SRT 1, G-SRT 2, G-SRT 3, G-SRT 4, G-SRT 5
- **Modeling Unit** — 4 days
- **P3 PROJECT** — 5 days
- **G4 Coordinate Geometry** — 15 days: G-GPE 4, G-GPE 5, G-GPE 6, G-GPE 7*
- **G5 Circles and Conics** — 15 days: G-C 1, G-C 2, G-C 5, G-GPE 1, G-GPE 2
- **G7 Geometric Measurement & Dimensions** — 15 days: G-GMD 1, G-GMD 3, G-GMD 4
- **G6 Trigonometric Ratios** — 15 days: G-SRT 6, G-SRT 7, G-SRT 8
- **M4 Capstone Geometric Modeling Project** — 10 days: G-MG 1*, G-MG 2*, G-MG 3*
- **P4 PROJECT** — 5 days

## Grade 11: Algebra Two

TRADITIONAL

- **A0 Intro Unit** — 5 days
- **A7 Exponential Functions** — 25 days: N-RN 1, N-RN 2, A-SSE 3c, A-SSE 4*, A-REI 11, F-IF 3, F-IF 7e, F-IF 8b, F-BF 2, F-BF 4a, F-LE 1, F-LE 1a, b, c, F-LE 2, F-LE 3, F-LE 4, F-LE 5
- **A8 Trigonometric Functions** — 15 days: F-IF 7e, F-TF 1, F-TF 2, F-TF 5, F-TF 8
- **A9 Functions** — 15 days: N-RN 1, N-RN 2, A-SSE 1, A-SSE 1a, A-SSE 1b, A-REI 11, F-IF 4, F-IF 7b, F-IF 7c, F-BF 1, F-BF 1a, b, F-BF 3, F-LE 3
- **Modeling Unit** — 8 days
- **P5 PROJECT** — 5 days
- **A10 Rational and Polynomial Expressions** — 20 days: N-RN 3, N-CN 1, N-CN 2, N-CN 7, A-SSE 1, A-SSE 1a, 1b, A-SSE 2, A-APR 1, A-APR 2, A-APR 3, A-APR 4, A-APR 6, A-REI 2, A-REI 11
- **P1 Probability** — 28 days: S-CP 1, S-CP 2, S-CP 3, S-CP 4, S-CP 5, S-CP 6, S-CP 7
- **S2 Statistics (Random Processes)** — 26 days: S-IC 1, S-IC 2, S-IC 3, S-IC 4, S-IC 5, S-IC 6, S-ID 4
- **Modeling Unit** — 4 days
- **P6 PROJECT** — 5 days

I N T E R N A T I O N A L

# INTEGRATED — Grade 9

- **I-0 Intro Unit** — 5 days
- **A1 Modeling with Functions** — 15 days: N-Q 1, N-Q 2, N-Q 3, F-IF 1, F-IF 2, F-IF 3, F-IF 4*, F-IF 5*, F-IF 9*, F-LE 1, F-LE 1a,1b, F-LE 3, F-LE 5
- **A2 Linear Functions** — 15 days: F-IF 6, F-IF 7a, F-IF 9, F-BF 1, F-BF 1a, F-BF 2, F-BF 4a, F-LE 1, F-LE 1a,1b
- **A3 Linear Equations & Inequalities in One Variable** — 15 days: A-REI 1, A-REI 3, A-REI 11*, A-CED 1, A-CED 3, A-CED 4
- **A4 Linear Equations & Inequalities in Two Variables** — 15 days: A-CED 2, A-CED 3, A-CED 4, A-REI 5, A-REI 6, A-REI 10, A-REI 12
- **Modeling Unit** — 4 days
- **P1 PROJECT** — 5 days
- **G0 Tools and Construction** — 10 days: G-CO 12, G-CO 13
- **G1 Basic Definitions & Rigid Motions** — 20 days: G-CO 1, G-CO 3, G-CO 2, G-CO 4, G-CO 5, G-CO 6, G-CO 7, G-CO 8
- **G2 Geometric Relationships & Properties** — 15 days: G-CO 9, G-CO 10, G-CO 11, G-C 3
- **Modeling Unit** — 4 days
- **S1 Statistics** — 30 days: S-ID 1, S-ID 2, S-ID 3, S-ID 5, S-ID 6, S-ID 6a, b, c, S-ID 7, S-ID 8, S-ID 9
- **P2 PROJECT** — 5 days

# INTEGRATED — Grade 10

- **I-0 Intro Unit** — 5 days
- **G4 Coordinate Geometry** — 15 days: G-GPE 4, G-GPE 5, G-GPE 6, G-GPE 7*
- **A5 Quadratic Functions** — 20 days: F-IF 4*, F-IF 5*, F-IF 6, F-IF 7, F-IF 7a*, F-IF 8, F-IF 8a, F-IF 9, F-BF 1, F-BF 1a, b, F-BF 3
- **A6 Quadratic Equations** — 26 days: N-CN 7, A-SSE 3a, A-SSE 3b, A-REI 4, A-REI 4a,b, A-REI 7
- **Modeling Unit** — 4 days
- **P3 PROJECT** — 5 days
- **G3 Similarity** — 20 days: G-SRT 1, G-SRT 2, G-SRT 3, G-SRT 4, G-SRT 5
- **G5 Circles and Conics** — 20 days: G-C 1, G-C 2, G-C 5, G-GPE 1, G-GPE 2
- **P1 Probability** — 28 days: S-CP 1, S-CP 2, S-CP 3, S-CP 4, S-CP 5, S-CP 6, S-CP 7
- **M4 Capstone Geometric Modeling Project** — 10 days: G-MG 1*, G-MG 2*, G-MG 3*
- **P4 PROJECT** — 5 days

# INTEGRATED — Grade 11

- **I-0 Intro Unit** — 5 days
- **G7 Geometric Measurement & Dimension** — 15 days: G-GMD 1, G-GMD 3, G-GMD 4
- **A7 Exponential Functions** — 25 days: N-RN 1, N-RN 2, A-SSE 3c, A-SSE 4*, A-REI 11, F-IF 3, F-IF 7e, F-IF 8b, F-BF 2, F-BF 4a, F-LE 1, F-LE 1a, b, c, F-LE 2, F-LE 3, F-LE 4, F-LE 5
- **G6 Trigonometric Ratios** — 15 days: G-SRT 6, G-SRT 7, G-SRT 8
- **Modeling Unit** — 8 days
- **P5 PROJECT** — 5 days
- **A8 Trigonometric Functions** — 15 days: F-IF 7e, F-TF 1, F-TF 2, F-TF 5, F-TF 8
- **A9 Functions** — 15 days: N-RN 1, N-RN 2, A-SSE 1, A-SSE 1a, A-SSE 1b, A-REI 11, F-IF 4, F-IF 7b, F-IF 7c, F-BF 1, F-BF 1a, b, F-BF 3, F-LE 3
- **A10 Rational and Polynomial Expressions** — 20 days: N-RN 3, N-CN 1, N-CN 2, N-CN 7, A-SSE 1, A-SSE 1a,1b, A-SSE 2, A-APR 1, A-APR 2, A-APR 3, A-APR 4, A-APR 6, A-REI 2, A-REI 11
- **S2 Statistics (Random Processes)** — 26 days: S-IC 1, S-IC 2, S-IC 3, S-IC 4, S-IC 5, S-IC 6, S-ID 4
- **Modeling Unit** — 4 days
- **P6 PROJECT** — 5 days

# Curriculum and Implementation Effects on High School Students' Mathematics Learning From Curricula Representing Subject-Specific and Integrated Content Organizations

Douglas A. Grouws, James E. Tarr, Óscar Chávez,
Ruthmae Sears, Victor M. Soria, and Rukiye D. Taylan
*University of Missouri*

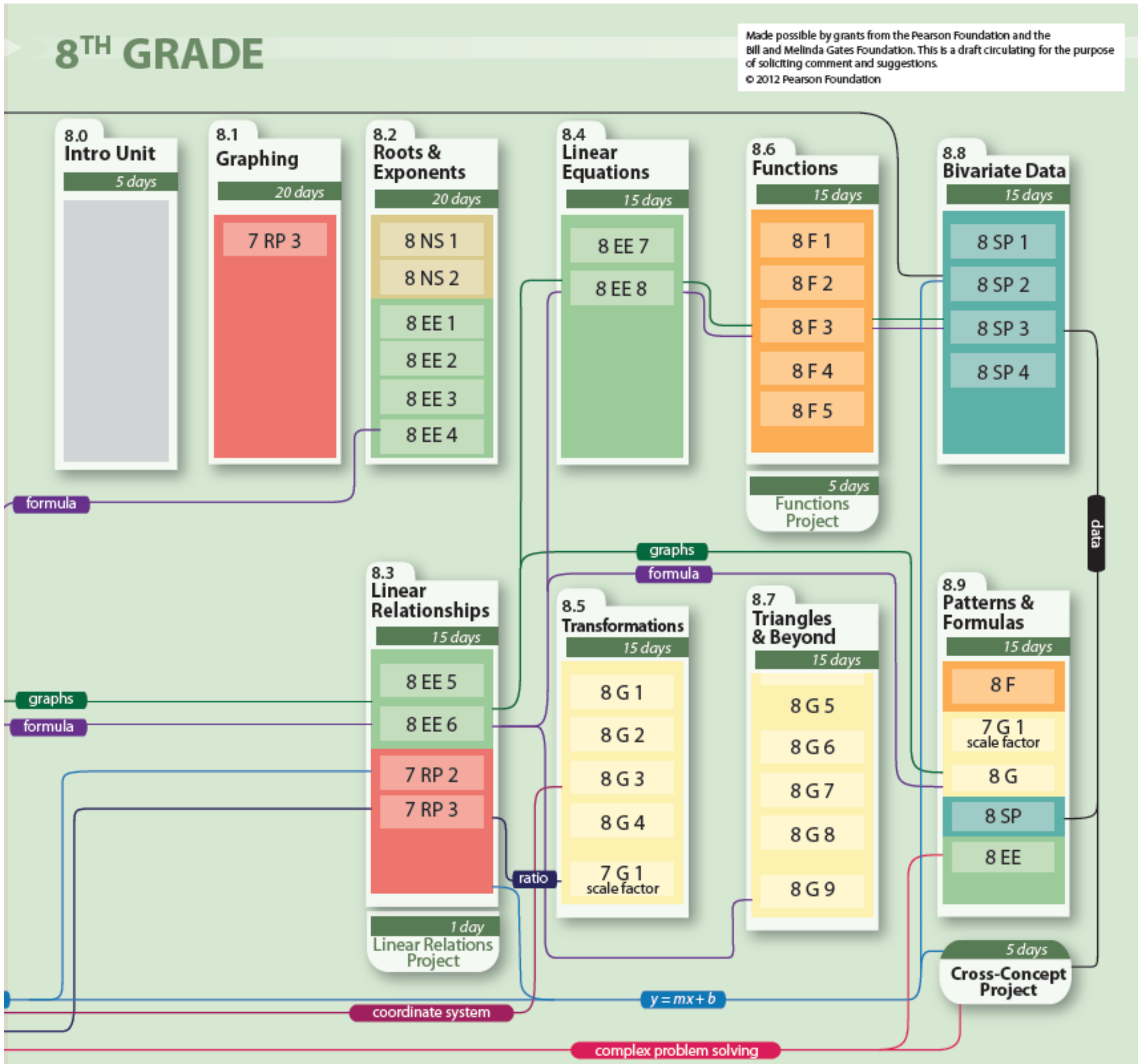
This study examined the effect of 2 types of mathematics content organization on high school students' mathematics learning while taking account of curriculum implementation and student prior achievement. The study involved 2,161 students in 10 schools in 5 states. Within each school, approximately 1/2 of the students studied from an integrated curriculum (Course 1) and 1/2 studied from a subject-specific curriculum (Algebra 1). Hierarchical linear modeling with 3 levels showed that students who studied from the integrated curriculum were significantly advantaged over students who studied from a subject-specific curriculum on 3 end-of-year outcome measures: Test of Common Objectives, Problem Solving and Reasoning Test, and a standardized achievement test. Opportunity to learn and teaching experience were significant moderating factors.

**Jason Zimba**
**co-Author CCSSM**

It is **incorrect** to say that algebra isn't covered until high school. There is a great deal of algebra in the 8th grade standards.

For example, students in grade 8 are expected to solve two simultaneous equations with two unknowns. I don't see a lack of rigor there. The standards actually invest heavily in algebra because of the way they focus so strongly on the prerequisites for algebra in the elementary grades.

# CCSSM 8th Grade are HS Standards




- Algebra/Functions 67%
- Geometry (Transformations and Triangle Proofs) 20%
- Bivariate Data 10%
- Cross-Concept Project 3%

# When do we Accelerate?????

# Where to Accelerate?

Can we live without understanding....

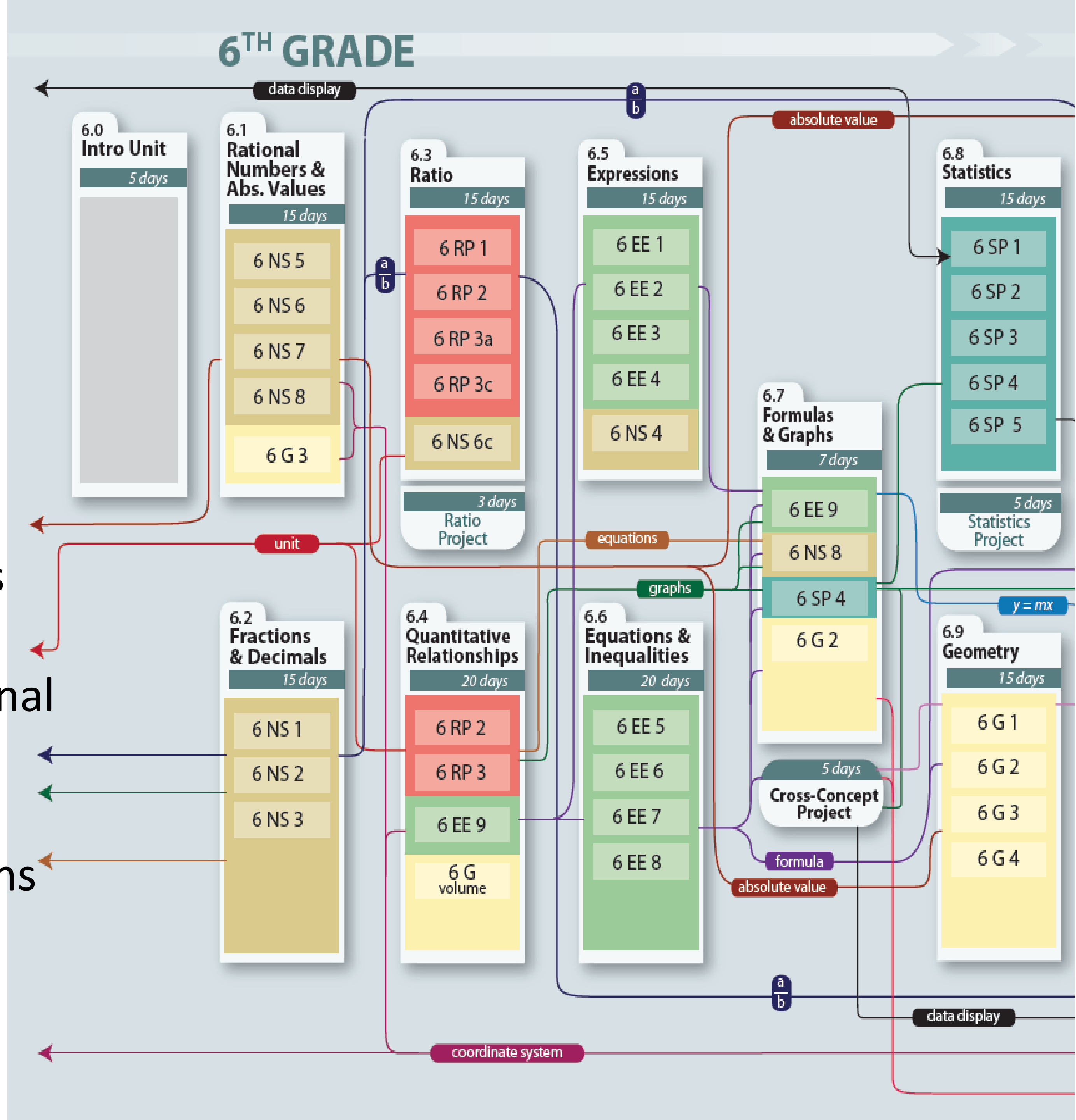
Integer and their operations

Division of Fractions

Ratio and proportional reasoning

Expression, Equations and Inequalities

Statistics



**Where to Accelerate?**

Can we live without understanding….

Properties of rational numbers, percents, discounts, markups, etc.

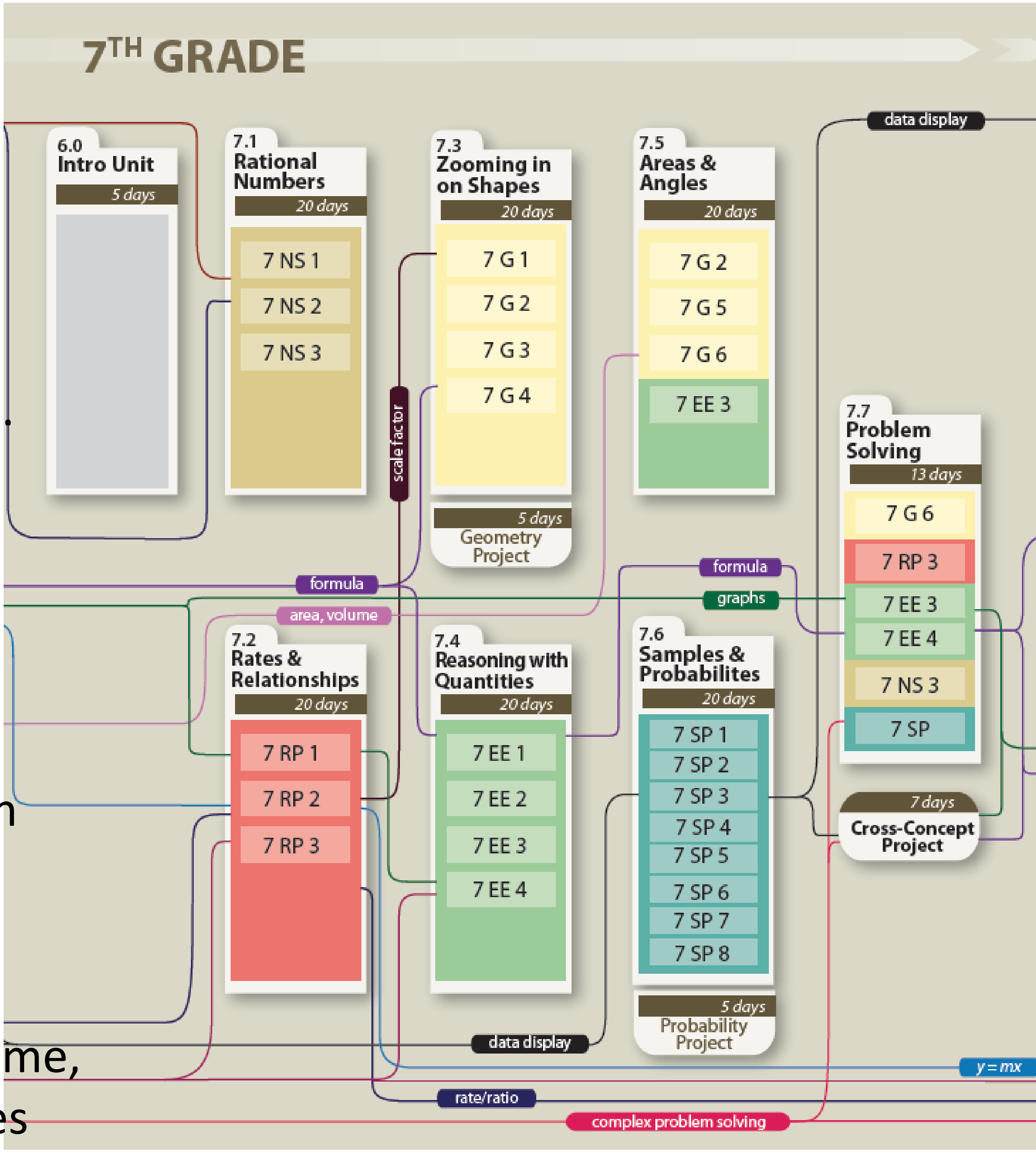
Rate and problems solving using rate

Similarity, proportional reasoning

Algebraic Modeling with Equations

Probability

Geometry: Angles, Volume, Surface Area, 3-D shapes



# When do we Accelerate?????

How will kids who are ready for advanced work accelerate to reach courses like calculus during high school?

**<u>Those are questions for policy, not for standards</u>**. The standards don't speak to this issue. Decisions about acceleration and ability grouping are still the purview of local districts, just as they've always been.

**Jason Zimba**
**co-Author CCSSM**

# Appendix A

COMMON CORE STATE STANDARDS FOR

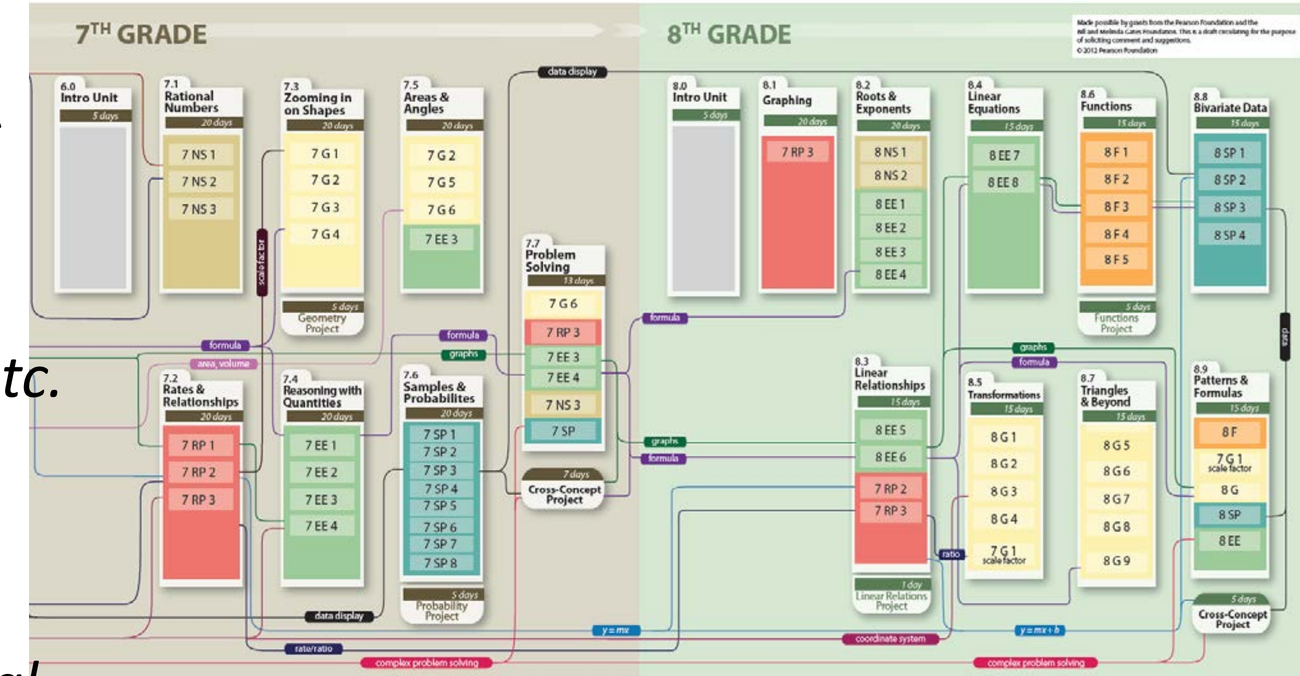
## Mathematics

Appendix A:

Designing High School Mathematics Courses Based on the Common Core State Standards

Brad Findell

# Accelerated Seventh Grade by Appendix A




*Properties of rational numbers, percents, discounts, markups, etc.*

*Rate and problems solving using rate*

*Similarity, proportional reasoning*

Algebraic Modeling with Equations

Probability

*Geometry: Angles, Volume, Surface Area, 3-D shapes*

**In Addition you have nearly all of the 8th grade CCSSM course in 7th (accept for 3 standard sets)**

Algebra/Functions (through Systems of Equations)

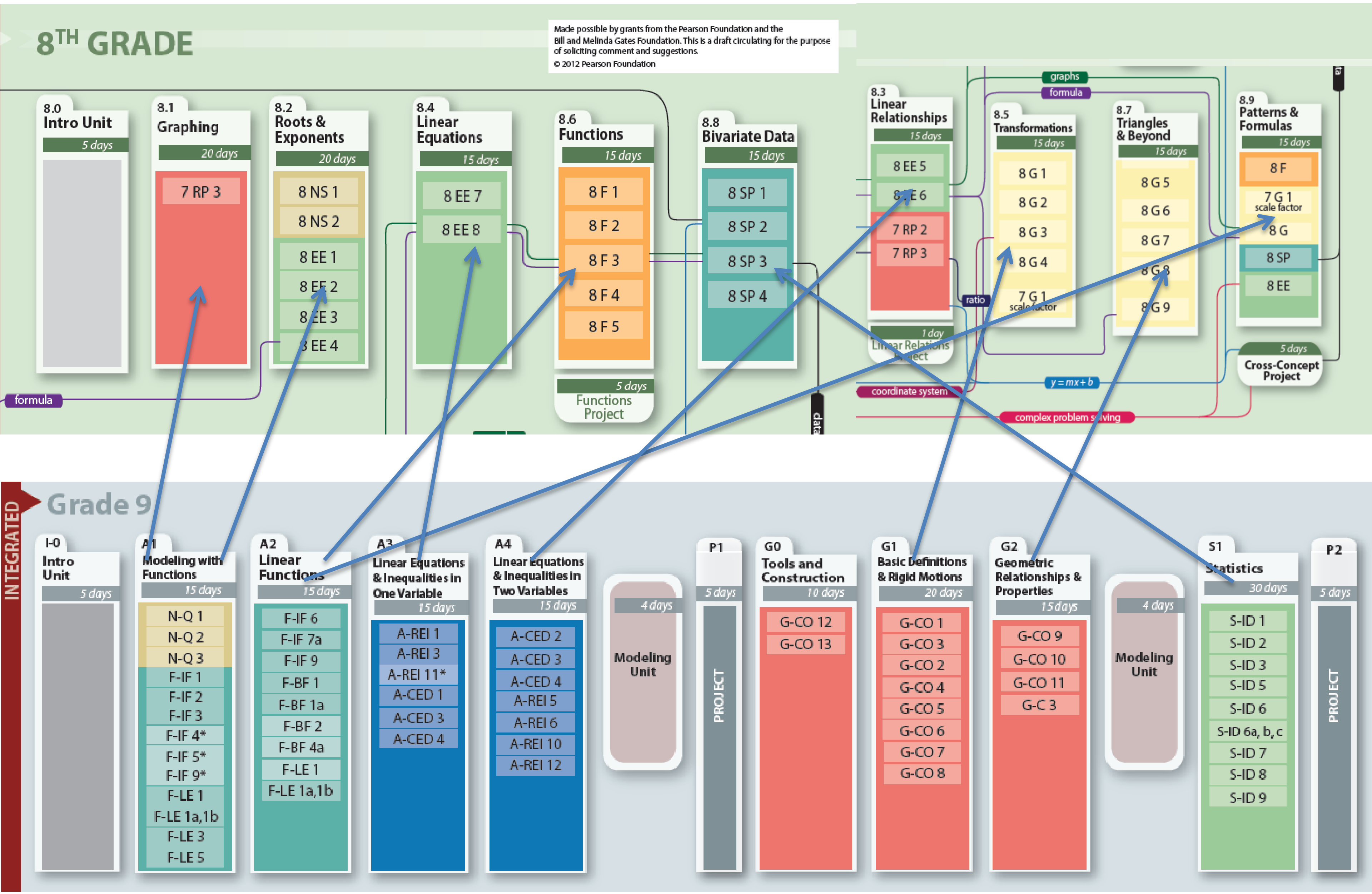
Geometry (Congruence and Similairty Triangle Proofs)

Statistical Inferences

# When do they Accelerate in Japan?

After 8th Grade!!!!!!!

# Where to Accelerate????



# When do we Accelerate?????


**The Only Reasonable Answer for Learning: 9th Grade!!!!**

# College Ready Sequence

| | | | | | | |
|---|---|---|---|---|---|---|
| $6^{th}$ Grade | $7^{th}$ Grade | $8^{th}$ Grade | Math 1<br>International | Math 2<br>International | Math 3<br>International | HS Year 4<br>Pre-Calculus<br>Math Analysis<br>AP Statistics<br>Finite Math |

# Accelerated Sequence

| | | | | | | |
|---|---|---|---|---|---|---|
| $6^{th}$ Grade | $7^{th}$ Grade | $8^{th}$ Grade | Math 2<br>International | Math 3<br>International | HS Year 4<br>Pre-Calculus<br>Math Analysis | AP Calculus |

# California Mathematics Framework: Possible Pathways to Calculus in 12th Grade

For clarity, "HS Course 1, 2 or 3" could refer to either the "traditional" high school pathway (Algebra 1, Geometry, Algebra 2) or "integrated" (Math 1, Math 2, Math 3).

1. ***Compacting in Middle School: Three CCSS courses in two years during grades 7 and 8***
   Decision point to accelerate: after grade 6

| Grade 6 students | Grade 7 students | Grade 8 students | Grade 9 students | Grade 10 students | Grade 11 students | Grade 12 students |
|---|---|---|---|---|---|---|
| CCSS 6 | CCSS 7 <u>and</u><br>CCSS 8, part 1 | CCSS 8, part 2 <u>and</u><br>CCSS HS Course 1 | CCSS HS Course 2 | CCSS HS Course 3 | CCSS HS Course 4 | AP Calculus |

2. ***Doubling Up in High School: Two CCSS courses during two class periods of math in grade 9***
   Decision point to accelerate: after grade 8

| Grade 6 students | Grade 7 students | Grade 8 students | Grade 9 students | Grade 10 students | Grade 11 students | Grade 12 students |
|---|---|---|---|---|---|---|
| CCSS 6 | CCSS 7 | CCSS 8 | 1st semester:<br>CCSS HS Course 1 | CCSS HS Course 3 | CCSS HS Course 4 | AP Calculus |
| | | | 2nd semester:<br>CCSS HS Course 2 | | | |

3. ***Compacting in High School: Three CCSS courses in two years during grades 9 and 10***
   Decision point to accelerate: after grade 8

| Grade 6 students | Grade 7 students | Grade 8 students | Grade 9 students | Grade 10 students | Grade 11 students | Grade 12 students |
|---|---|---|---|---|---|---|
| CCSS 6 | CCSS 7 | CCSS 8 | CCSS HS Course 1<br><u>and</u><br>CCSS HS Course 2A | CCSS HS Course 2B<br><u>and</u><br>CCSS HS Course 3 | CCSS HS Course 4 | AP Calculus |

4. ***Enhanced Pathway in High School: STEM High School Courses 1, 2, and 3 will include the advanced CCSS (+) pre-calculus standards***
   Decision point to accelerate: after grade 8 (STEM – Science, Technology, Engineering, and Mathematics)

| Grade 6 students | Grade 7 students | Grade 8 students | Grade 9 students | Grade 10 students | Grade 11 students | Grade 12 students |
|---|---|---|---|---|---|---|
| CCSS 6 | CCSS 7 | CCSS 8 | Enhanced (STEM)<br>CCSS HS Course 1 | Enhanced (STEM)<br>CCSS HS Course 2 | Enhanced (STEM)<br>CCSS HS Course 3 | AP Calculus |

5. ***Pre-Calculus Summer Bridge Pathway: After completing Courses 1, 2 and 3, students can take a summer course in preparation for Calculus***
   Decision point to accelerate: after grade 11

| Grade 6 students | Grade 7 students | Grade 8 students | Grade 9 students | Grade 10 students | Grade 11 students | Grade 12 students |
|---|---|---|---|---|---|---|
| CCSS 6 | CCSS 7 | CCSS 8 | CCSS HS Course 1 | CCSS HS Course 2 | CCSS HS Course 3 | AP Calculus |

Summer<br>Pre-Calculus

# Discussion Questions to Consider

- How are the current math pathways enabling your students to be college and career ready? What opportunities and challenges do students face?
- What are the merits and demerits of the *traditional* US high school pathway versus an *international* pathway?
- What data should we consider in evaluating our current system? What do we already have available and what would need to be researched or tracked?
- What would it take to really change your current pathway system?
  - What are political implications?
  - What articulation would be required in your vertical feeder system?
  - Who would need to be educated and how?
  - How would current students be phased into a new pathway system?
  - What would it take for your institution to be successful in this change process?